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Meta Theorem
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This program tests whether a sentence of Boolean Logic is a theorem, a contradiction, or a contingency. A theorem is a sentence that is true for all variable assignments, whereas a contradiction is a sentence that is always false; a contingency is a sentence that can be true sometimes or false.

There are 6 logical operations: 'and', 'or', 'xor', 'implies', 'iff', 'not'. Enter these using the following symbols: & | * > = !. Logical variables are represented by single letters, single digits, @, or #. The logical constants 'true' and 'false' are represented by + and -. Therefore, this program can handle sentences with up to 64 variables. This program evaluates all operations from left to right; use parentheses to change the way a logical sentence is evaluated. Blank spaces are parsed out.

For example, the logical sentence '(P implies Q) iff ((not P) or Q)' would be entered as (P > Q) = (!P | Q).

The first thing that this program does is convert the sentence into postfix form. Postfix form is more efficient especially if the sentence is to be evaluated many times. For example, the sentence (P & (Q | R)) = ((P & Q) | (P & R)) would be converted to P Q R | & P Q & P R & | =. During the conversion process, the syntax of the sentence is checked for errors. Then, the program goes through the truth table of the sentence, starting from the all true case, and proceeding to all false. It searches for 2 different variable assignments such that one makes the sentence true (model), and the other makes the sentence false (counterexample). If after going through all $2^n$ lines of the truth table, the program doesn't find a model and a counterexample, then either the sentence is a theorem if only models were found, or it's a contradiction if only counterexamples were found. Of course, if both a model and a counterexample were found, the sentence is a contingency. Here, n is the number of variables in the sentence.

While the applet is running, the percentage of progress through the truth table is displayed. Scrolling through the truth table is possible with the Model and Counterexample buttons; the Reset button returns the pointer to the first line of the truth table. It is advisable to reset the pointer when changing scroll modes.